$\mathbb{Z}_h$ and $\mathbb{D}_h$-reduced derivative NLS in the limit $h \to \infty$.

Vladimir S. Gerdjikov [1]

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
Acad. Georgi Bonchev Str., Block 8, 1113 Sofia, Bulgaria
E-mail: `vgerdjikov@math.bas.bg`

Special types of derivative nonlinear Schrödinger equations with $\mathbb{Z}_h$ and $\mathbb{D}_h$–reductions are analyzed [1], see also [2]

$$i\frac{\partial \psi_k}{\partial t} + \gamma \frac{\partial}{\partial x}\left(\operatorname{cotan}\frac{\pi k}{h} \cdot \psi_{k,x} + i\sum_{p=1}^{h-1} \psi_p\psi_{k-p} \right) = 0, \tag{1}$$

where $k = 1, 2, \dots, h-1$ γ is a constant and the index $k-p$ should be understood modulus h, $\psi_0 = \psi_h = 0$. They allow also the involutions:

$$\text{a)} \qquad \psi_k = -\psi_k^*, \qquad \gamma = -\gamma^*, \qquad \text{b)} \quad \psi_k = \psi_{h-k}^*, \qquad \gamma = \gamma^*. \tag{2}$$

The limit $h \to \infty$ leads to $2+1$ dimensional models. Indeed, if

$$u(x,t,y) = \sum_{j=1}^{h-1} \psi_j(x,t) \exp\left(\frac{2\pi i j y}{h} \right). \tag{3}$$

Then (2b) means that $u(x,t,y)$ is real-valued. Retaining only the leading term of cotan $(\pi k/h)$ and rescaling $x = \xi/h$, $\tau = t/h$ in (1) we get:

$$u_{y\tau} + \gamma u_{\xi\xi} + \gamma (u)^2_{\xi y} = 0, \tag{4}$$

which is a Kadomtsev-Petviashvilli-like equation without the dispersive term.

References

[1] V. S. Gerdjikov. *Z_N–reductions and new integrable versions of derivative nonlinear Schrödinger equations.* In: Nonlinear evolution equations: integrability and spectral methods, Ed. A. P. Fordy, A. Degasperis, M. Lakshmanan, Manchester University Press, (1981), p. 367–372.

[2] V. S. Gerdjikov. *Derivative Nonlinear Schrödinger Equations with $\mathbb{Z}_N$ and $\mathbb{D}_N$–Reductions.* Romanian Journal of Physics, **58**, Nos. 5-6, 573-582 (2013).

[1] Associate collaborator of Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72 Tsarigradsko chausee, Sofia 1784, Bulgaria,