Göttingen Workshop on Integrable Systems 2016

List of registered participants/speakers

- **Olga Assainova** (Institut de Mathématiques de Bourgogne, Dijon, France)
- **Dorothea Bahns** (Mathematical Institute, University of Göttingen, Germany)
- **Julia Bernatska** (National University of Kyiv-Mohyla Academy, Kiev, Ukraine)
  Title of talk: *An integrable system on trigonal curve: solutions in terms of multivariate Abelian functions*
  Abstract: We consider an integrable analog of MKdV-system constructed on a coadjoint orbit in sl(3)-related loop algebra (introduced by P. Holod, 1997). The system possesses a trigonal spectral curve, and its dynamics is described in terms of the corresponding multivariate Abelian functions. The latter is easily seen from the scheme of separation of variables, leading to the Jacobi inversion problem. Following V. Buchstaber & D. Leykin (Funct. An., 2002, 2004, 2008), we apply Weierstrass' approach to construct solutions in terms of multivariate Abelian functions.
- **Oleg Chalykh** (Department of Applied Mathematics, University of Leeds, UK)
  Title of talk: *Quantum Lax pairs for Calogero-Moser problem*
  Abstract: A quantum Lax pair for the Calogero-Moser problem was first considered in type A_n by Hikami, Ujino and Wadati (1992) and also later for other root systems by Bordner, Manton and Sasaki (2000) who comment on a close resemblance of the Lax matrix and the Dunkl operator. I will provide an explanation why the two objects are indeed closely related. This is joint work with M. Feigin (Glasgow).
- **Jen-Hsu Chang** (Chung Cheng Institute of Technology, Department of Computer Science and Information Engineering, National Defense University, Taiwan)
  Title of talk: *Asymptotic analysis of rogue waves in the KP-I equation*
  Abstract: Inspired by the works of Y. Ohta and J. Yang, one constructs the rogue waves solutions in the KP-I using the Gram determinants. It is shown that the number of lumps will depend on the real roots of Wronskian of some orthogonal polynomials for the asymptotic behaviors. In particular, one can prove that all the lumps are on a vertical line when time approaches $-\infty$, and then they will be on a horizontal line when time approaches ∞, i.e., there is a $\pi/2$ rotation after interactions.
- **Oleksandr Chvartatskyi** (Mathematical Institute, University of Göttingen, and Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany)
  Title of talk: *Binary Darboux transformations in bidifferential calculus and systems with self-consistent sources*
  Abstract: In the framework of bidifferential calculus we apply a universal binary Darboux transformation result to generate families of exact solutions of integrable systems. The latter includes Peregrine and Akhmediev breathers of the focusing NLS equation, multipole lump solutions of the KP I equation, dromions and solitoffs of the Davey-Stewartson system. Further examples contain a generalization of Hirota's bilinear difference equation and a two-dimensional Toda lattice equation. Matrix generalizations are treated as well. We also reveal the origin and

structure of self-consistent source extensions of integrable equations from the perspective of binary Darboux transformations. They arise via a deformation of the potential that is central in this solution generating method. This talk is based on a joint work with Aristophanes Dimakis and Folkert Müller-Hoissen.

- **Aristophanes Dimakis** (University of the Aegean, Chios, Greece)
  Title of talk: *Bäcklund transformations in bidifferential calculus*
  Abstract: Bäcklund transformations (BT) are studied in the framework of bidifferential calculi. We define higher BT, and we discuss compositions of BT and permutability. We apply these tools in the example of noncommutative KdV where some equations related to KdV are recovered.
- **Victor Enolski** (National University "Kyiv Mohyla Academy", Ukraine)
  Title of talk: *Charge two monopole within the Nahm Ansatz*
  Abstract: The ADHMN-construction (Atiyah-Drinfeld-Hitchin-Manin-Nahm) of the Higgs field and gauge fields for a nonabelian monopole reduces to solving a Weyl equation, a linear ODE with "potentials" given by the so called Nahm data. Even in the case of charge two, where the Nahm data is expressible in terms of elliptic functions, the analytical expressions for the monopole fields are still unknown. We overcome the problem using a lesser known, comparatively to the ADHMN construction, Nahm Ansatz which implementation leads to complete description of monopole fields in terms of algebraic functions and incoplete elliptic integrals.
- **Yuri Fedorov** (Departament de Matemàtica Aplicada I ETSEIB-UPC, Barcelona, Spain)
  Title of talk: *A shortcut to the Kovalevskaya curve*
  Abstract: There have already been numerous studies and interpretations of the celebrated separation of variables in the integrable top of S. Kovalevskaya. In the talk it will be shown how the known Kovalevskaya curve of separation can be obtained, by a simple one-step transformation, from the spectral curve of the corresponding Lax representation found by Reimann and Semonov-Tian-Shanski. The algorithm works for the general constants of motion of the top and is based on W. Barth's description of Prym varieties via pencils of genus 3 curves. It can be used for separation of variables of various generalizations of the Kovalevskaya top.
- **Evgeny Ferapontov** (Department of Mathematical Sciences, Loughborough University, UK)
  Title of talk: *Homogeneous third-order Hamiltonian operators*
  Abstract: I will address the classification of homogeneous third-order Hamiltonian operators of differential-geometric type. Operators of this form appear in applications as Hamiltonian structures of the associativity equations, and possess a number of unexpected connections with projective differential geometry. Based on the correspondence with quadratic line complexes, a complete list of such operators with n≤5 components is obtained.
  [1] E.V. Ferapontov, M.V. Pavlov and R.F. Vitolo, Projective-geometric aspects of homogeneous third-order Hamiltonian operators, J. Geom. Phys. 85 (2014) 16-28; arXiv:1401.7235
  [2] E.V. Ferapontov, M.V. Pavlov and R.F. Vitolo, Towards the classification of homogeneous third-order Hamiltonian operators, IMRN (2016), doi:10.1093/imrn/rnv369; arXiv:1508.02752.
- **Vladimir S. Gerdjikov** (Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria)
  Title of talk: *$\mathbb{Z}_h$ and $\mathbb{D}_h$-reduced derivative NLS in the limit $h \to \infty$*
  Abstract

- **Francesco Giglio** (Northumbria University, Newcastle upon Tyne, UK)
  Title of talk: *Integrable extended van der Waals model*
  Abstract: An extension of the celebrated van der Waals equation of state has been obtained as the solution to a suitable C-integrable nonlinear conservation law which is equivalent to the first principle of thermodynamics. The gas-liquid phase transition appears in the thermodynamic limit which is equivalent to the inviscid limit and leads to a Riemann-Hopf type equation. The small viscous parameter is played by the inverse of the Avogadro's number. The construction and the phenomenology of the model is discussed and a comparison between the equation of state obtained in the thermodynamic limit and two of the most popular empirical equations of state is shown.
- **Matteo Gorgone** (University of Messina, Messina, Italy)
  Title of talk: *Decoupling of first order quasilinear systems of PDEs*
  Abstract: We focus on the decoupling problem of general quasilinear first order systems in two independent variables. We consider the case of homogeneous and autonomous systems as well as the one of nonhomogeneous and/or nonautonomous systems. Necessary and sufficient conditions for the partial or full decoupling of the systems at hand are provided. Remarkably, the proof involves only the properties of eigenvalues and eigenvectors of the coefficient matrix, and it is constructive in the sense that it gives the differential constraints whose integration leads to the decoupling transformation. Some applications of physical interest are also discussed.
- **Georgi Grahovski** (Department of Mathematical Sciences, University of Essex, UK)
  Title of talk: *On the N-wave equations with PT-symmetry*
  Abstract: We will present extensions of N-wave systems with PT-symmetry. The types of (nonlocal) reductions leading to integrable equations invariant with respect to P- (spatial reflection) and T- (time reversal) symmetries is described. The corresponding constraints on the fundamental analytic solutions and the scattering data are derived. Based on examples of 3-wave (related to the algebra sl(3,C)) and 4-wave (related to the algebra so(5,C)) systems, the properties of different types of 1- and 2-soliton solutions are discussed. It is shown that the PT symmetric 3-wave equations may have regular multi-soliton solutions for some specific choices of their parameters. This talk is based on a joint work with Vladimir Gerdjikov and Rossen Ivanov.
- **Gerard Helminck** (Korteweg-de Vries Institute for Mathematics, Amsterdam, The Netherlands)
  Title of talk: *Strict versions of matrix hierarchies*
  Abstract: In this talk we consider a different splitting of the matrix pseudo differential operators, giving rise to new compatible systems of Lax equations in this algebra. We will discuss their structure, give a geometric construction of solutions and treat various reductions. This leads to strict versions of well-known systems like the AKNS- and multicomponent KP hierarchy.
- **Karima Khusnutdinova** (Department of Mathematical Sciences, Loughborough University, UK)
  Title of talk: *Ring waves in a stratified fluid over a shear flow: cKdV-type equation and singular solutions for wavefronts*
  Abstract: We consider long ring waves in a stratified fluid in the presence of a depth-dependent parallel shear flow (e.g., ocean waves over currents) and derive a 2+1-dimensional cylindrical Korteweg-de Vries-type equation for the amplitudes of surface and internal waves. The wavefront is described by the singular solution (envelope of the general solution) of a nonlinear first-order

differential equation. We find the singular solution for the case of a two-layer fluid with the piecewise-constant current and discuss some properties of the waves. This is joint work with Xizheng Zhang.

- **Christian Klein** (Institut de Mathématiques de Bourgogne, Dijon, France)
  Title of talk: *Numerical study of 2+1 dimensional nonlinear dispersive PDEs*
  Abstract: We present several numerical studies of solutions to PDEs from the family of nonlinear Schrödinger and Kortweg-de Vries equations. We study the formation of dispersive shocks and of potential blow-ups in the solutions. A universality conjecture for the break-up of the solutions is presented.
- **Boris Konopelchenko** (Physics Department, Università del Salento, Lecce, Italy)
  Title of talk: *Confluences, Jordan forms and regularizations of hydrodynamic type systems*
  Abstract: Novel class of integrable hydrodynamic type systems which govern the dynamics of critical points of confluent Lauricella type functions defined on the finite dimensional Grassmannians Gr(2,n) is constructed. It is shown that in general such systems are given by non-diagonalizable quasi-linear systems with normal Jordan blocks. Connection of the systems corresponding to the deepest confluence with the problems of the change of type transitions for the quasilinear systems of mixed type and possible regularizations of the Burgers-Hopf equation is discussed.
- **Olaf Lechtenfeld** (Institute for Theoretical Physics, University of Hannover, Germany)
  Title of talk: *The tetrahexahedric angular Calogero model*
  Abstract: The spherical reduction of the rational Calogero model (of type A_{n-1} and after removing the center of mass) is considered as a maximally superintegrable quantum system, which describes a particle on the (n−2)-sphere subject to a very particular potential. We present a detailed analysis of the simplest non-separable case, n=4, whose potential is singular at the edges of a spherical tetrahexahedron. A complete set of independent conserved charges and of Hamiltonian intertwiners is constructed, and their algebra is elucidated. They arise from the ring of polynomials in Dunkl-deformed angular momenta, by classifying the subspaces invariant and antiinvariant under all Weyl reflections, respectively.
- **Franco Magri** (Dipartimento di Matematica e Applicazioni, Università degli Studi di Milano-Bicocca, Milano, Italy)
  Title of talk: *Examples of Lenard complexes*
  Abstract: I intend to discuss a couple of examples of Lenard complexes on a Haantjes manifold, which are of interest to the theory of integrable systems.
- **Giovanni Manno** (Politecnico di Torino, Dipartimento di Scienze Matematiche (DISMA) "Giuseppe Luigi Lagrange", Torino, Italy)
  Title of talk: *Hydrodynamic integrability as a geometric feature of Lagrangian Grassmannians — part I*
  Abstract: In this first part we introduce a geometric framework for second-order PDEs, based on contact manifolds and their Lagrangian Grassmannian bundles, also known as their first prolongations. Thus, scalar second-order PDEs can be interpreted as hypersurfaces in such bundles. The peculiar geometry of a Lagrangian Grassmannian allows us to recognise special classes of hypersurfaces: the "simplest ones" are the Monge-Ampère equations, which can be

geometrically interpreted as hyperplane sections (via Plücker embedding) or, more physically, as completely exceptional PDEs in the sense of Lax.

- **Giovanni Moreno** (Institute of Mathematics of the Polish Academy of Sciences, Warsaw, Poland)
  Title of talk: *Hydrodynamic integrability as a geometric feature of Lagrangian Grassmannians — part II*
  Abstract: In this second part we focus on the geometry of Lagrangian Grassmannian, understood as a projective variety through the so-called Plücker embedding. A tangible interpretation of the Plücker embedding space is that its hyperplanes are in one-to-one correspondence with the Monge-Ampère equations. After reviewing the notion of dual space of a Lagrangian Grassmannian, we show that the dual of the Plücker embedding space is naturally stratified by it and its singular loci. In particular, there is a stratum which corresponds to the Monge-Ampère equations that are integrable by the method of hydrodynamic reductions.
- **Michal Marvan** (Mathematical Institute, Opava, Czech Republic)
  Title of talk: *Nonlocal conservation laws of the constant astigmatism equation*
  Abstract: We construct four linearly independent hierarchies of nonlocal conservation laws of the constant astigmatism equation and explore how they behave under the reciprocal transformations.
- **Vladimir B. Matveev** (Université de Bourgogne, Dijon, France)
  Title of talk: *Rogue Waves in 1+1 and 2+1 integrable systems, related with AKNS and KP-I hierarchies*
  Abstract: The discovery of multi-rogue wave (MRW) solutions of the focusing NLS equation, made in 2010 (Eur. Phys. J., Special topics 185 (2010) 247-258), drastically improved a vision of the links between rogue waves and the theory of integrable systems. The multiple rogue wave solutions of the NLS equation can be described by means of a Wronskian determinant representation with a very simple structure. This structure allows to relate them with multi-rogue waves solutions of the KP-I equation via a remarkable relation, which we call NLS-KP-I correspondence. We provide arguments showing that "extreme" rogue waves in 2+1 dimensional models occur as a result of the collision of a certain number of "simple" rogue waves. Moreover, we will show that it is possible to extend the basic formulas, which we used in the NLS case, to obtain the MRW rational or quasi-rational solutions of any rank, for all equations of the reduced AKNS (RAKNS) hierarchy. We also show that a (for the NLS equation) well known scaling invariance and Galilean invariance property of solutions can be extended, with appropriate modifications, to the whole AKNS hierarchy and its reduced and deformed versions.
- **Antonio Moro** (Northumbria University, Newcastle upon Tyne, UK)
  Title of talk: *On the integrable structure and thermodynamics of “complex" networks*
  Abstract: A large variety of real world systems can be naturally modelled by networks, i.e. graphs whose nodes represent the components of a system linked (interacting) according to specific statistical rules. A complex network is realised via a graph of nontrivial topology typically constituted by a large number of nodes/links. Fluid and magnetic models in Physics are just two among the many classical examples of systems which can be modelled by using simple or complex networks. In particular "extreme" conditions (thermodynamic regime), networks, just like fluid and magnetic models, exhibit a critical behaviour intended as a drastic change of state due to a continuous change of thermodynamic parameters. Using an approach to thermodynamics, recently introduced

to describe a general class of van der Waals type models and magnetic systems in mean field approximation, we analyse the integrable structure of corresponding networks and use the theory of conservation laws to provide an analytical description of the system outside and inside the critical region.

- **Oleg Morozov** (AGH University of Science and Technology, Cracow, Poland)
  Title of talk: *Deformed cohomologies of symmetry pseudo-groups and zero-curvature representations of differential equations*
  Abstract: I will talk about a relation between deformed cohomologies of symmetry pseudo-groups and zero-curvature representations of differential equations. Examples will include the potential Khokhlov-Zabolotskaya equation, the Boyer-Finley equation and the exceptional case of the rmdKP equation.
- **Folkert Müller-Hoissen** (Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany)
- **Francesco Oliveri** (University of Messina, Messina, Italy)
  Title of talk: *Nonlinear first order PDEs reducible to first order autonomous systems polynomially homogeneous in the derivatives*
  Abstract: A theorem is proved providing necessary and sufficient conditions enabling one to map a nonlinear system of first order partial differential equations, polynomial in the derivatives, to an equivalent autonomous first order system polynomially homogeneous in the derivatives. The result is intimately related to the symmetry properties of the source system. The proof, which involves the use of canonical variables associated to the admitted Lie point symmetries, is constructive. Some examples concerned with Monge systems to which the theorem can be applied are considered.
- **Maxim Pavlov** (Novosibirsk State University, Russia)
  Title of talk: *The Gibbons-Tsarev System and Isomonodromic Deformations*
  Abstract: In this talk we consider the remarkable Gibbons-Tsarev system, which describes multi-component hydrodynamic reductions of integrable three-dimensional quasilinear equations of second order. We show that this system possesses infinitely many solutions, determined by isomonodromic deformations of integrable dispersive systems. Description of these solutions will be presented in this talk.
- **Vladimir Sokolov** (Russian Academy of Sciences, Landau Institute for Theoretical Physics, Moscow, Russia)
  Title of talk: *Integrable cosmological potentials*
  Abstract: The Friedman-Lemaitre-Robertson-Walker spatially flat cosmologies with a scalar field are described by Hamiltonians of the form $H = p_1^2 - p_2^2 + \exp(q_1)\, V(q_2)$ with the Friedman constraint H=0. We desribe all such Hamiltonians that have additional integrals of first or second degree in momenta on the level H=0. New integrable potentials with integrals of third degree are found.
- **Taras Skrypnyk** (Università degli Studi di Milano-Bicocca, Milano, Italy)
  Title of talk: *Separation of variables in the Clebsch model*
  Abstract: We perform a variable separation in completely anisotropic Clebsch model. We bring the corresponding equations of motion, written in the separated coordinates, to the Abel-type form and explicitly reconstruct the initial dynamical variables of the model via the coordinates and momentas

of separation.

- **Nikola Stoilov** (Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany)
  Title of talk: *Camassa-Holm version of WDVV associativity equations for N = 3*
  Abstract: We present a scheme to understand the WDVV associativity equations in the case N = 3 as a dispersive deformation of a (1+1)-dimensional hydrodynamic type system. The WDVV equations themselves can be written as a hydrodynamic-type system, which however does not allow Riemann invariants. It possesses finitely many conservation laws and a Lax pair, thus it is integrable by inverse scattering rather than hodograph transformation, typical for quasilinear systems. Using a non-local change of coordinates, depending on a parameter and extensively using the bi-Hamiltonian structure of WDVV, we present a dispersive system which has infinitely many dispersive terms and a dispersionless limit, possesses Riemann invariants, and when written in non-evolutionary form can be viewed as a three component Camassa-Holm type equation.
- **Nikolai Ustinov** (Kaliningrad branch of Moscow State University of Railway Communications, Kaliningrad, Russia)
  Title of talk: *Extremely Short Vector Solitons in Anisotropic Media*
  Abstract: The nonlinear dynamics of extremely short electromagnetic pulses in a biaxial crystal under the conditions of conical refraction is considered. The Ostrovsky-Vakhnenko equation is derived as governing equation under certain conditions. This equation is integrable by means of the inverse scattering transformation method and connected with the Tzitzeica equation through a change of variables. The properties of the soliton solutions are discussed.
- **Alexander Veselov** (Department of Mathematical Sciences, Loughborough University, UK)
  Title of talk: *Logarithmic Frobenius structures and theory of hyperplane arrangements*
  Abstract: The logarithmic Frobenius structures correspond to a special class of solutions of the famous Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations. There are examples related to Coxeter configurations and their restrictions, but the general classification is still an open problem. I will discuss some relations of this problem with the theory of hyperplane arrangements, including holonomy Lie algebras and logarithmic vector fields. The talk is based on a joint work with M. Feigin.